$$x^4-(a^2+b^2+2c^2)x^2+4abcx-c^2(a^2+b^2-c^2)=0 \;\ldots\ldots\ldots\ldots (5).$$

Restoring numbers in (5), we have

$$x^4-2518x^2+14400x-22419=0.$$

Solving this equation by Horner's Method, we find $x=47.145$ feet, nearly.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Virginia. All contributions to this department should be sent to him.

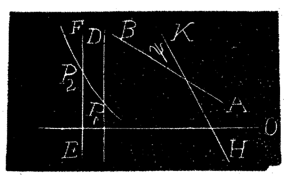
SOLUTIONS OF PROBLEMS.

58. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

A line passes through a fixed point and rotates uniformly about this point. Another line passes through a point which moves uniformly along the arc of a given curve and rotates uniformly about this point. Develop a method for finding the locus of intersection of these two lines. Apply to case of circle and straight line.

II. Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

Let O be the origin, P_3 the fixed point, its coördinates being (r_3, θ_3), and let AB be a given position of line through P_3. Let $P_1(r_1, \theta_1)$ be position of point on curve and CD the line through it, both corresponding to the position AB of other line. Also let HK be position of AB revolved through an $\angle\psi$, and let $P_2(r_2, \theta_2)$ and EF be the corresponding position of P_1 and CD.



Let $r=f(\theta)$ be equation to curve P_1P_2. Let η=the angle made by AB, and η_1 the one made by CD with a polar axis. Let a=angular rate of revolution of AB, and na of CD.

$\therefore\ \angle$ between CD and $EF=n\psi$.

Let b=linear rate of movement of P_1. Then $\psi/a=P_1P_2/b \ldots\ldots\ldots$ (1).

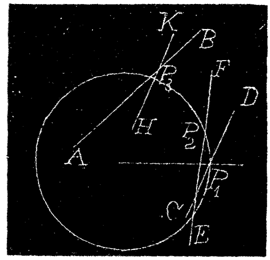
Equation to KH is $r=[r_3\sin(\eta+\psi-\theta_3)]/\sin(\eta+\psi-\theta) \ldots\ldots\ldots\ldots$ (2).

Equation to EF is $r=(r_2\sin(\eta_1+n\psi-\theta_2)]/\sin(\eta_1+n\psi-\theta) \ldots\ldots\ldots\ldots$ (3).

By integration,

$$P_1P_2=\int_{\theta_1}^{\theta_2}\sqrt{r^2+[(dr/d\theta)]^2}\,d\theta=\int_{\theta_1}^{\theta_2}\sqrt{[f(\theta)]^2+[f'(\theta)]^2}\,d\theta\ldots\ldots\ldots\ldots(4),$$

which gives P_1P_2 in terms of θ_2, θ_1 being known. Then substitute from (4) in (1) to get ψ in terms of θ_2. Substitute this value, and also $f(\theta_2)$ for r_2 in (2) and (3). Then by eliminating (θ_2) we have resulting the equation to the locus of the intersecting of the lines. The solution depends on our ability to integrate (4). Now if the given lines are not straight, it is evident that the only changes are in equations (2) and (3). These may be derived from the equations to the lines in original position by a method of transformation of coördinates. For example, the equation to HK may be derived from that to AB by revolving the pole and polar axis about P_3 through an angle equal to ψ and in the opposite direction. If the given curve is a circle and the lines straight, the problem can be definitely solved as follows:



Transform coördinates so that center of circle shall be pole and OP_1 the the polar axis. Then $r=f(\theta)$ becomes $r=c$.

Let $(r_1, \theta_1)(r_2, \theta_2)(r_3, \theta_3)$ represent the new coördinates of points P_1, P_2, P_3, respectively.

Then $r_1=r_2=c$. $\theta_1=0$. $P_1P_2=c\theta_2$. From (1) $\psi=ac\theta_2/b$.

(2) becomes $r=[r_3\sin(\eta+ac\theta_2/b-\theta_3)]/\sin(\eta+ac\theta_2/b-\theta)\ldots\ldots\ldots\ldots(5)$.

(3) becomes $r=[c\sin(\eta_1+nac\theta_2/b-\theta_2)]/\sin(\eta_1+nac\theta_2/b-\theta)\ldots\ldots\ldots(6)$.

(5) can be solved for θ_2 and the result can be substituted in (6), giving the equation required. Then if desired the coördinates can be again transformed to the original form.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

43. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Two weights P and Q rest on the concave side of a parabola whose axis is horizontal, and are connected by a string, length l, which passes over a smooth peg at the focus, F. [*Bowser's Analytical Mechanics*, page 54.]